

# Information on the inflaton field from the spectrum of relic gravitational waves

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## Abstract

After a review of a traditional analysis, it is shown a variation of a more recent treatment on the spectrum of relic gravitational waves (GWs). Then, a connection between the two different treatments will be analysed. Such a connection permits to obtain an interesting equation for the inflaton field. This equation gives a value that agrees with the slow roll condition on inflation.

## 1 Introduction

The scientific community aims in a first direct detection of GWs in next years (for the current status of GWs interferometers see [1]) confirming the indirect, Nobel Prize Winner, proof of Hulse and Taylor [2].

Detectors for GWs will be important for a better knowledge of the Universe and either to confirm or to rule out, in an ultimate way, the physical consistency of General Relativity, eventually becoming an observable endorsing of Extended Theories of Gravity, see [3] for details.

It is well known that an important potential source of gravitational radiation is the relic stochastic background of GWs, see [4] for a recent review. The potential existence of such a relic stochastic background arises from general assumptions. In fact, it derives from a mixing between basic principles of classical theories of gravity and of quantum field theory. The zero-point quantum oscillations, which produce relic GWs, are generated by strong variations of the gravitational field in the early universe. Then, the detection of relic GWs is the only way to learn about the evolution of the very early universe, up to the

bounds of the Planck epoch and the initial singularity [4, 5, 6, 7]. The importance of this *gravity's rainbow* in cosmological scenarios has been discussed in an elegant way in [8].

The model derives from the inflationary scenario for the early universe [6, 9], which is tuned in a good way with the WMAP data on the Cosmic Background Radiation (CBR) (in particular exponential inflation and spectral index  $\approx 1$  [10, 11]). Recently, the analysis has been adapted to extended theories of gravity too [4, 12, 13].

After a review of a traditional analysis on the spectrum of relic GWs following [7, 12, 13], in this paper a variation of a more recent analysis that uses a conformal treatment is performed. A connection between the two different treatments will be also analysed. This connection permits to obtain an interesting equation for the inflaton field. This equation gives a value that agrees with the slow roll condition on inflation [9, 20].

For a sake of simplicity, in this paper natural units are used, i.e.  $8\pi G = 1$ ,  $c = 1$  and  $\hbar = 1$ . The Latin indices run from 1 to 3, the Greek ones from 0 to 3.

Considering a stochastic background of GWs, it can be characterized by a dimensionless spectrum [7, 12, 13]

$$\Omega_{gw}(f) \equiv \frac{1}{\rho_c} \frac{d\rho_{gw}}{d\ln f}, \quad (1)$$

where

$$\rho_c \equiv \frac{3}{8} H_0^2 \quad (2)$$

is the (actual) critical density energy,  $\rho_c$  of the Universe,  $H_0$  the actual value of the Hubble expansion rate and  $d\rho_{gw}$  the energy density of relic GWs in the frequency range  $f$  to  $f + df$ .

The more recent values for the spectrum can be found in [12, 13, 14, 15].

## 2 A review of the traditional analysis on the relic GWs spectrum

Following the traditional analysis in [7, 12, 13] it will be assumed that the universe is described by a simple cosmology in two stages, an inflationary De Sitter phase and a radiation dominated phase. Then, the line element of the spacetime is given by [7, 12, 13]

$$ds^2 = a^2(\eta)[-d\eta^2 + d\underline{x} + h_{\mu\nu}(\eta, \underline{x})dx^\mu dx^\nu], \quad (3)$$

and the calculation will be performed for the exponential inflationary model that agrees with the WMAP data [10, 11].

In the De Sitter phase ( $\eta < \eta_1$ ) the equation of state is  $P = -\rho = \text{const}$ , the scale factor is  $a(\eta) = \eta_1^2 \eta_0^{-1} (2\eta_1 - \eta)^{-1}$  and the Hubble constant is given by  $H_{ds} = \eta_0/\eta_1^2$  [7, 12, 13].

In the radiation dominated phase ( $\eta > \eta_1$ ) the equation of state is  $P = \rho/3$ , the scale factor is  $a(\eta) = \eta/\eta_0$  and the Hubble constant is given by  $H = \eta_0/\eta^2$  [7, 12, 13].

Expressing the scale factor in terms of comoving time defined by [16]

$$dt = a(t)d\eta \quad (4)$$

one gets

$$a(t) \propto \exp(H_{ds}t) \quad (5)$$

during the De Sitter phase, and

$$a(t) \propto \sqrt{t} \quad (6)$$

during the radiation dominated phase. In order to obtain a solution for the horizon and flatness problems one needs [9]

$$\frac{a(\eta_0)}{a(\eta_1)} > 10^{26}. \quad (7)$$

This value is the equivalent of 60 e-foldings, where one e-folding is defined as the amount of time for  $a$  to grow by a factor of  $e$  [9, 20].

The relic GWs are the weak perturbations  $h_{\mu\nu}(\eta, \underline{x})$  of the metric (3). By considering, for example, the + plus polarization of GWs [16], in terms of the conformal time  $\eta$  it is [7, 12, 13]

$$h_+(\eta, \underline{k}, \underline{x}) = X(\eta) \exp(\underline{k} \times \underline{x}), \quad (8)$$

where  $\underline{k}$  is a constant wave vector.

By putting  $Y(\eta) = a(\eta)X(\eta)$  and with the standard linearized calculation in which the connections (i.e. the Cristoffel coefficients), the Riemann tensor, the Ricci tensor and the Ricci scalar curvature are found, from Friedman linearized equations one gets that the function  $Y(\eta)$  satisfies the equation [7, 12, 13]

$$\frac{d^2 Y}{d\eta^2} + (k^2 - \frac{1}{a} \frac{d^2 a}{d\eta^2}) Y = 0 \quad (9)$$

Clearly, this is the equation for a parametrically perturbed oscillator.

The solutions of eq. (9) give the solutions for the function  $X(\eta)$ , that can be expressed in terms of elementary functions simple cases of half integer Bessel or Hankel functions [7, 12, 13] in both of the inflationary and radiation dominated eras:

for  $\eta < \eta_1$

$$X(\eta) = \frac{a(\eta_1)}{a(\eta)} [1 + H_{ds}\omega^{-1}] \exp -ik(\eta - \eta_1), \quad (10)$$

for  $\eta > \eta_1$

$$X(\eta) = \frac{a(\eta_1)}{a(\eta)} [\alpha \exp -ik(\eta - \eta_1) + \beta \exp ik(\eta - \eta_1)], \quad (11)$$

where  $\omega = k/a$  is the angular frequency of the wave (that is function of the time because of the constancy of  $k = |\underline{k}|$ ),  $\alpha$  and  $\beta$  are time-independent constants which can be obtained by demanding that both  $X$  and  $dX/d\eta$  are continuous at the boundary  $\eta = \eta_1$  between the inflationary and the radiation dominated eras of the cosmological expansion. With this constrain it is

$$\alpha = 1 + i \frac{\sqrt{H_{ds} H_0}}{\omega} - \frac{H_{ds} H_0}{2\omega^2} \quad (12)$$

$$\beta = \frac{H_{ds} H_0}{2\omega^2} \quad (13)$$

In eqs. (12) and (13)  $\omega = k/a(\eta_0)$  is the angular frequency that would be observed today, and  $H_0 = 1/\eta_0$  is the Hubble expansion rate that would be observed today. These calculations are called *Bogoliubov coefficient methods* [7, 12, 13].

In inflationary scenarios both of classical and macroscopic perturbations are damped out by inflation. Thus, the minimum allowed level of fluctuations is that required by quantum uncertainty principle. The choice of the solution (10) corresponds precisely to such a De Sitter vacuum state [7, 12, 13]. Then, if the period of inflation was long enough, the observable properties of the Universe today should be indistinguishable from the properties of a Universe started in the De Sitter vacuum state.

In the radiation dominated phase the eigenmodes which describe particles are the coefficients of  $\alpha$ , and the eigenmodes which describe antiparticles are the coefficients of  $\beta$  [7, 12, 13]. Thus, the number of created particles of angular frequency  $\omega$  in the radiation dominated phase is given by

$$N_\omega = |\beta_\omega|^2 = \left( \frac{H_{ds} H_0}{2\omega^2} \right)^2. \quad (14)$$

In this way, the expression for the energy density of the stochastic relic gravitons background in the frequency interval  $(\omega, \omega + d\omega)$  can be written down like

$$d\rho_{gw} = 2\omega \left( \frac{\omega^2 d\omega}{2\pi^2} \right) N_\omega = \frac{H_{ds}^2 H_0^2}{4\pi^2} \frac{d\omega}{\omega} = \frac{H_{ds}^2 H_0^2}{4\pi^2} \frac{df}{f}. \quad (15)$$

Eq. (15) can be re-written in terms of the present day and the De Sitter energy-density of the universe. For the Hubble expansion rates it is

$$\begin{aligned} H_0^2 &= \frac{\rho_c}{3} \\ H_{ds}^2 &= \frac{\rho_{ds}}{3}. \end{aligned} \quad (16)$$

Then, the spectrum is given by [7, 12, 13]

$$\Omega_{gw}(f) = \frac{1}{\rho_c} \frac{d\rho_{gw}}{d \ln f} = \frac{f}{\rho_c} \frac{d\rho_{gw}}{df} = \frac{16}{9} \frac{\rho_{ds}}{\rho_{Planck}}, \quad (17)$$

and the introduced Planck density  $\rho_{Planck}$  is normalized in our units.

Actually, the calculation works for a very simplified model that does not include the matter dominated era. If this era is also included the redshift has to be considered. An enlightening computation in [7] gives

$$\Omega_{gw}(f) = \frac{16}{9} \frac{\rho_{ds}}{\rho_{Planck}} (1 + z_{eq})^{-1}, \quad (18)$$

for the waves which at the time in which the Universe was becoming matter dominated had a frequency higher than  $H_{eq}$ , the Hubble constant at that time. This corresponds to frequencies  $f > (1 + z_{eq})^{1/2} H_0$ , where  $z_{eq}$  is the redshift of the Universe when the matter and radiation energy density were equal. The redshift correction in equation (18) is needed because the Hubble parameter, which is governed by Friedman equations, should be different from the observed one  $H_0$  for a Universe without matter dominated era.

At lower frequencies the spectrum is given by [7, 12, 13]

$$\Omega_{gw}(f) \sim f^{-2}. \quad (19)$$

The results (17) and (18) cannot be applied in all the range of physical frequencies. In fact, for waves with frequencies less than  $H_0$  today, the notion of energy density has no sense, because the wavelength becomes longer than the scale of the Universe. In the same way, at high frequencies there is a maximum frequency above which the spectrum drops to zero rapidly [7, 12, 13].

In the above calculation, the simple assumption that the phase transition from the inflationary to the radiation dominated epoch is instantaneous has been implicitly made. In the real Universe this process occurs over some time scale  $\Delta\tau$ , and above a frequency

$$f_{max} = \frac{a(t_1)}{a(t_0)} \frac{1}{\Delta\tau}, \quad (20)$$

which is the red shifted rate of the transition,  $\Omega_{gw}$  drops rapidly. The two cutoffs (at low and high frequencies) to the spectrum guarantee that the total energy density of the relic GWs is finite [7, 12, 13].

### 3 The conformal analysis by using the inflaton field

Now, a variation of a recent treatment [17], that used a conformal analysis, will be considered.

The GW-equations in the TT gauge are [16]

$$\square h_i^j = 0. \quad (21)$$

Matter perturbations do not appear in (21) since scalar and tensor perturbations do not couple with tensor perturbations in Einstein equations [17].

The more general scalar-tensor action in 4 dimensions is given by [12]

$$S = \int d^4x \sqrt{-g} [\varphi R - \frac{\omega(\varphi)}{\varphi} g^{\mu\nu} \varphi_{;\mu} \varphi_{;\nu} - W(\varphi) + \mathcal{L}_m], \quad (22)$$

and in the framework of inflationary theories the scalar field  $\varphi$  works like inflaton [9, 20].

One can perform the conformal transformation [17]

$$\tilde{g}_{\alpha\beta} = e^{2\Phi} g_{\alpha\beta} \quad (23)$$

where the conformal rescaling

$$e^{2\Phi} = \varphi \quad (24)$$

has been chosen.  $\Phi$  is the “conformal scalar field”. The difference with reference [17] is that in such a work the analysis concerned  $f(R)$  theories, while we focus our attention on the inflaton scalar field  $\varphi$ .

By applying the conformal transformation (23) to the action (22) the conformal equivalent Hilbert-Einstein action

$$A = \int \frac{1}{2k} d^4x \sqrt{-\tilde{g}} [\tilde{R} + L(\Phi, \Phi_{;\alpha})], \quad (25)$$

is obtained.  $L(\Phi, \Phi_{;\alpha})$  is the conformal scalar field contribution derived from

$$\tilde{R}_{\alpha\beta} = R_{\alpha\beta} + 2(\Phi_{;\alpha} \Phi_{;\beta} - g_{\alpha\beta} \Phi_{;\delta} \Phi^{;\delta} - \frac{1}{2} g_{\alpha\beta} \Phi^{;\delta}_{;\delta}) \quad (26)$$

and

$$\tilde{R} = e^{-2\Phi} (R - 6\square\Phi - 6\Phi_{;\delta} \Phi^{;\delta}). \quad (27)$$

In the rescaled action (25) the matter contributions have not been considered because our interaction with GWs concern the linearized theory in vacuum.

In [17] it has been shown that  $h_i^j$  is a conformal invariant and that the d'Alembert operator transforms as

$$\tilde{\square} = e^{-2\Phi} (\square + 2\Phi^{;\alpha} \partial_{;\alpha}). \quad (28)$$

Thus, the background is changing while the tensor wave amplitude is fixed.

In order to study the cosmological stochastic background, the operator (28) has to be specified for a Friedman-Robertson-Walker metric [17], obtaining

$$\ddot{h}_+ + (3H + 2\dot{\Phi})\dot{h}_+ + k^2 a^{-2} h_+ = 0, \quad (29)$$

being  $\square = \frac{\partial}{\partial t^2} + 3H \frac{\partial}{\partial t}$ ,  $a(t)$  the scale factor and  $k$  the wave number.

Considering the conformal time  $d\eta = dt/a$ , eq. (29) reads

$$\frac{d^2}{d\eta^2}h_+ + 2\frac{\gamma'}{\gamma}\frac{d}{d\eta}h_+ + k^2h_+ = 0, \quad (30)$$

where  $\gamma = ae^\Phi$ . Inflation means that  $a(t) = a_0 \exp(Ht)$  and then  $\eta = \int dt/a = 1/(aH)$  and  $\frac{\gamma'}{\gamma} = -\frac{1}{\eta}$ . The exact solution of (30) is [17]

$$h_+(\eta) = k^{-3/2}\sqrt{2/k}[C_1(\sin k\eta - \cos k\eta) + C_2(\sin k\eta + \cos k\eta)]. \quad (31)$$

Inside the  $1/H$  radius it is  $k\eta \gg 1$ . Furthermore, considering the absence of GWs in the initial vacuum state, only negative-frequency modes are present and then the adiabatic behavior is [17]

$$h_+ = k^{1/2}\sqrt{2/\pi}\frac{1}{aH}C \exp(-ik\eta). \quad (32)$$

At the first horizon crossing ( $aH = k$  at  $t = 10^{-22}$  second after the Initial Singularity, see [7]), the averaged amplitude  $A_h = (k/2\pi)^{3/2}h_+$  of the perturbations is

$$A_h = \frac{1}{2\pi^2}C \quad (33)$$

when the scale  $a/k$  grows larger than the Hubble radius  $1/H$ , the growing mode of evolution is constant, i.e. “frozen” [17]. This situation corresponds to the limit  $-k\eta \ll 1$  in equation (31).

The amplitude  $A_h$  of the wave is preserved until the second horizon crossing after which it can be observed, in principle, as an anisotropy perturbation of the CBR [10, 11]. It can be shown that  $\frac{\delta T}{T} \leq A_h$  is an upper limit to  $A_h$  since other effects can contribute to the background anisotropy [18]. Then, it is clear that the only relevant quantity is the initial amplitude  $C$  in equation (32) which is conserved until the re-enter. Such an amplitude directly depends on the fundamental mechanism generating perturbations that depends on the inflaton scalar field which generates inflation.

Considering a single monochromatic GW, its zero-point amplitude is derived through the commutation relations [17]

$$[h_+(t, x), \pi_h(t, y)] = i\delta^3(x - y) \quad (34)$$

calculated at a fixed time  $t$ .

As it is [17]

$$\pi_h = e^{2\Phi}a^3\dot{h}_+, \quad (35)$$

equation (34) reads

$$[h_+(t, x), \dot{h}_+(y, y)] = i\frac{\delta^3(x - y)}{e^{2\Phi}a^3} \quad (36)$$

and the fields  $h_+$  and  $\dot{h}_+$  can be expanded in terms of creation and annihilation operators

$$h_+(t, x) = \frac{1}{(2\pi)^{3/2}} \int d^3k [h_+(t)e^{-ikx} + h_+^*(t)e^{ikx}] \quad (37)$$

$$\dot{h}_+(t, x) = \frac{1}{(2\pi)^{3/2}} \int d^3k [\dot{h}_+(t)e^{-ikx} + \dot{h}_+^*(t)e^{ikx}]. \quad (38)$$

The commutation relations in conformal time are then [17]

$$[h_+ \frac{d}{d\eta} h_+^* - h_+^* \frac{d}{d\eta} h] = i \frac{8\pi^3}{e^{2\Phi} a^3}. \quad (39)$$

Inserting (32) and (33), it is  $C = \sqrt{2}\pi^2 H e^{-\Phi}$  where  $H$  and  $\Phi$  are calculated at the first horizon crossing and then

$$A_h = \frac{\sqrt{2}}{2} H e^{-\Phi}, \quad (40)$$

which means that the amplitude of GWs produced during inflation directly depends on the inflaton field being  $\Phi = \frac{1}{2} \ln \varphi$ . Explicitly, it is

$$A_h = \frac{H}{\sqrt{2\varphi}}, \quad (41)$$

thus, we have found a relation that links directly the amplitude of relic GWs with the inflaton scalar field  $\varphi$  which generates inflation:

$$\varphi = \frac{H^2}{2A_h^2}. \quad (42)$$

## 4 The inflaton field and the slow roll condition on inflation

It is well known that the requirement for inflation, which is  $p = -\rho$  [9, 20], can be approximately met if one requires  $\dot{\varphi} \ll V(\varphi)$ , where  $V(\varphi)$  is the potential density of the field. This leads to the so called *slow-roll approximation* (SRA), which provides a natural condition for inflation to occur [9, 20]. The constraint on  $\dot{\varphi}$  is assured by requiring  $\ddot{\varphi}$  to be negligible. With such a requirement, the slow-roll parameters are defined (in natural units) by [9, 20]

$$\begin{aligned} \epsilon(\varphi) &\equiv \frac{1}{2} \left( \frac{V'(\varphi)}{V(\varphi)} \right)^2 \\ \eta(\varphi) &\equiv \frac{V''(\varphi)}{V(\varphi)}. \end{aligned} \quad (43)$$

Then, the SRA requirements are [9, 20]:



$$\epsilon \ll 1 \quad (44)$$

$$|\eta| \ll 1,$$

that are satisfied when it is [9, 20]

$$\varphi \gg M_{Planck}, \quad (45)$$

where the Planck mass, which is  $M_{Planck} \simeq 2.177 * 10^{-5}g$  in ordinary units and  $M_{Planck} = 1$  in natural units [16], has been introduced [9, 20].

Now, by using a connection between the two treatments on relic GWs, i.e. the traditional one and the conformal one, we show that the condition (45) is satisfied.

Let us start by recalling the equation for the characteristic amplitude  $h_C$ , see Equation 65 in [19],

$$h_C(f) \simeq 1.26 * 10^{-18} \left( \frac{1\text{Hz}}{f} \right) \sqrt{h_{100}^2 \Omega_{gw}(f)}. \quad (46)$$

This equation gives a value of the amplitude of the relic GWs stochastic background in function of the spectrum in the frequency range of ground based detectors [19]. Such a amplitude is also the strain applied on the detector's arms [19]. Such a range is given by the interval  $10Hz \leq f \leq 10KHz$  [1].

Using eq. (17) eq. (46) becomes

$$h_C(f) \simeq 1.26 * 10^{-18} \left( \frac{1\text{Hz}}{f} \right) \sqrt{h_{100}^2 \frac{16}{9} \frac{\rho_{ds}}{\rho_{Planck}}}. \quad (47)$$

The mean value of this quantity will be  $\simeq A_h$ , thus, by using eq. (41) it is

$$\frac{H}{\sqrt{2}\varphi} \simeq \frac{1.26 * 10^{-18} \sqrt{h_{100}^2 \frac{16}{9} \frac{\rho_{ds}}{\rho_{Planck}}} \int_{10}^{10000} f^{-1} df}{\int_{10}^{10000} df}, \quad (48)$$

that, by computing the integrals in the range  $10Hz \leq f \leq 10KHz$  and recalling that  $h_{100} \approx 0.74$  [9, 10, 11] and that for GUT energy-scale of inflation it is [7, 12, 13]

$$\frac{\rho_{ds}}{\rho_{Planck}} \approx 10^{-12}, \quad (49)$$

gives

$$\frac{H}{\sqrt{2}\varphi} \simeq 8.2 * 10^{-28}. \quad (50)$$

By restoring ordinary units and recalling that  $H \simeq 10^{22}Hz$  at the first horizon crossing [7], at the end it is

$$\varphi \simeq 7 * 10^4 g. \quad (51)$$

This value agrees with the slow roll condition on inflation. In fact, the condition (45) is surely satisfied being  $M_{Planck} \simeq 2.177 * 10^{-5}g$  in ordinary units.

## 5 Conclusions

After reviewing a traditional analysis on the relic GWs spectrum, in this paper a variation of a more recent analysis that uses a conformal treatment has been discussed. A connection between the two different treatments has been analysed too. This connection permitted to obtain an interesting equation for the inflaton field. This equation gave a value that agrees with the slow roll condition on inflation.

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